

General Certificate of Education
January 2007
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Thursday 18 January 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 Use the mid-ordinate rule with four strips of equal width to find an estimate for

$$\int_1^5 \frac{1}{1 + \ln x} dx, \text{ giving your answer to three significant figures.} \quad (4 \text{ marks})$$

- 2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. (4 marks)

- 3 The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \text{ for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \text{ for real values of } x, x \neq -1$$

- (a) Find the range of f . (2 marks)

- (b) The inverse of g is g^{-1} .

(i) Find $g^{-1}(x)$. (3 marks)

(ii) State the range of g^{-1} . (1 mark)

- (c) The composite function gf is denoted by h .

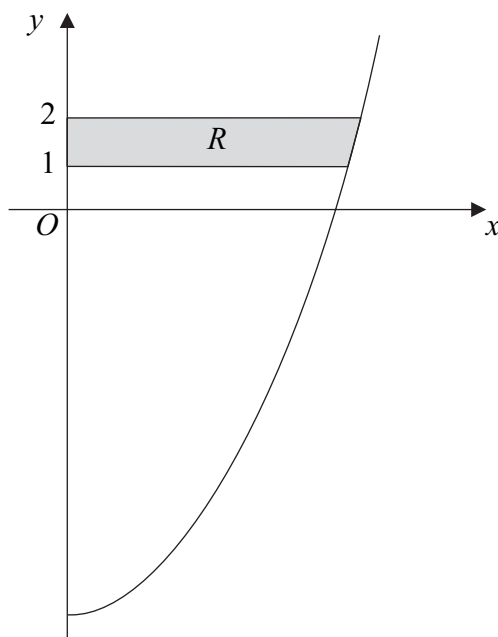
(i) Find $h(x)$, simplifying your answer. (2 marks)

(ii) State the greatest possible domain of h . (1 mark)

4 (a) Use integration by parts to find $\int x \sin x \, dx$. (4 marks)

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2 + 5} \, dx$. (4 marks)

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geq 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y -axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the y -axis. (4 marks)

5 (a) (i) Show that the equation

$$2 \cot^2 x + 5 \operatorname{cosec} x = 10$$

can be written in the form $2 \operatorname{cosec}^2 x + 5 \operatorname{cosec} x - 12 = 0$. (2 marks)

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. (3 marks)

(b) Hence, or otherwise, solve the equation

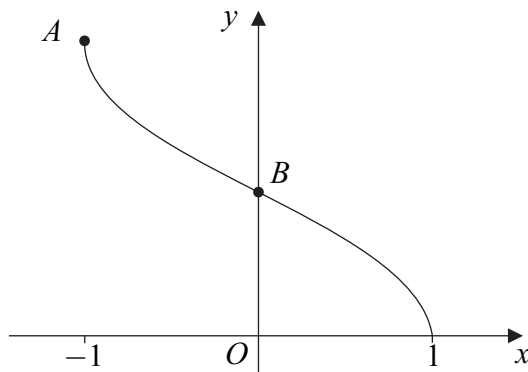
$$2 \cot^2(\theta - 0.1) + 5 \operatorname{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$.

(3 marks)

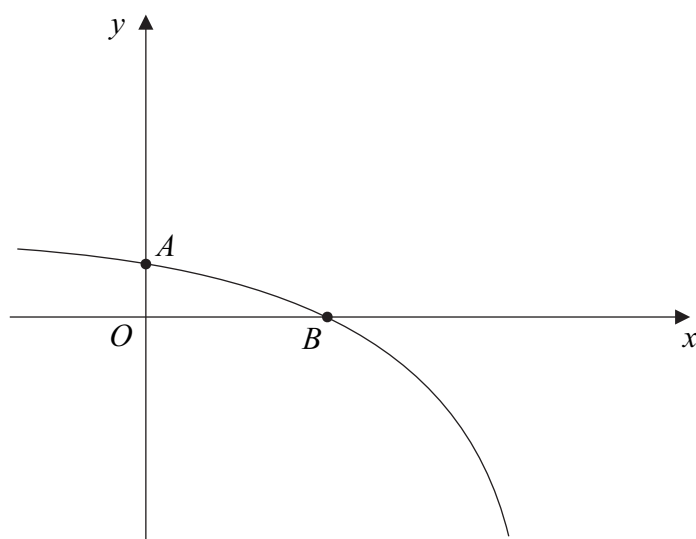
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- 6 (a) Find $\frac{dy}{dx}$ when:
- (i) $y = (4x^2 + 3x + 2)^{10}$; (2 marks)
- (ii) $y = x^2 \tan x$. (2 marks)
- (b) (i) Find $\frac{dx}{dy}$ when $x = 2y^3 + \ln y$. (1 mark)
- (ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point (2,1). (3 marks)
- 7 (a) Sketch the graph of $y = |2x|$. (1 mark)
- (b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. (3 marks)
- (c) Solve $4 - |2x| = x$. (3 marks)
- (d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. (2 marks)
- 8 The diagram shows the curve $y = \cos^{-1} x$ for $-1 \leq x \leq 1$.



- (a) Write down the exact coordinates of the points A and B . (2 marks)
- (b) The equation $\cos^{-1} x = 3x + 1$ has only one root. Given that the root of this equation is α , show that $0.1 \leq \alpha \leq 0.2$. (2 marks)
- (c) Use the iteration $x_{n+1} = \frac{1}{3}(\cos^{-1} x_n - 1)$ with $x_1 = 0.1$ to find the values of x_2 , x_3 and x_4 , giving your answers to three decimal places. (3 marks)

- 9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y -axis at the point A and the x -axis at the point B .



- (a) (i) Find $\int (4 - e^{2x}) dx$. (2 marks)
- (ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x}) dx = 4 \ln 2 - \frac{3}{2}$. (2 marks)
- (b) (i) Write down the y -coordinate of A . (1 mark)
- (ii) Show that $x = \ln 2$ at B . (2 marks)
- (c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B . (4 marks)
- (d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y -axis. (3 marks)

END OF QUESTIONS

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